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Reg. No. : ....

Name : ....

Third Semester B.Tech. Degree Examination, November 2014
(2013 Scheme)
13.303 : DISCRETE STRUCTURES (FR)

Time : 3 Hours Max. Marks : 100

PART – A

Answer **all** questions. **Each** question carries **2** marks.

1. Show that $\neg(P \wedge Q) \rightarrow (\neg P \vee (\neg P \vee Q)) \Leftrightarrow (\neg P \vee Q)$.

2. What is indirect method of Proof ? Using the same show that $\neg(P \wedge Q)$ follows from $\neg P \wedge \neg Q$.

3. Differentiate between a statement and a statement function.

4. If a Relation 'R' is transitive, then prove that its inverse R^{-1} is also transitive.

5. Let P = {2, 3, 6, 12, 24, 36}. The relation "$\leq$" be such that $x \leq y$; if x divides y. Draw the Hasse diagram of (P, $\leq$).

6. Define the symmetric difference between two sets.

7. Prove that the inverse of an element in a group is unique.

8. What is meant by Ring with zero divisors ? Explain.

9. Differentiate between simple and elementary path in a graph.

10. Prove that, in a distributive lattice, if an element has complement then this complement is unique. **(10×2=20 Marks)**

P.T.O.

PART – B

Answer **any one** question from **each** Module.

Module – I

11. a) Without constructing the truthtable, prove that

$(\neg P \wedge (\neg Q \wedge R)) \vee (Q \wedge R) \vee (P \wedge R) \Leftrightarrow R$. 7

b) Using indirect method, prove that

$S \to \neg Q, S \vee R, \neg R, \neg R \rightleftarrows Q \Rightarrow \neg P$. 7

c) Check the validity of the following statements

"All integers are rational numbers

Some integers are powers of 2.

Therefore, some rational numbers are powers of 2". 6

12. a) Show that

$((P \vee Q) \wedge \neg(\neg P \wedge (\neg Q \vee \neg R))) \vee (\neg P \wedge \neg Q) \vee (\neg P \wedge \neg R)$ is a tautology. 10

b) Show that from

i) $(\exists x)[F(x) \wedge S(x)] \to \forall y[M(y) \to W(y)]$

ii) $(\exists y)[M(y) \wedge \neg W(y)]$, the conclusion

$(\forall x)(F(x) \to \neg S(x))$ follows. 10

Module – II

13. a) Construct a formula for the sum of first 'n' positive odd numbers. Prove the same using mathematical induction. 5

b) Let R and S be two relations on a set of positive integers I.

$R = \{< x, 2x > / x \in I\} S = \{< x, 7x > / x \in I\}$. Find $R \circ S$, $R \circ R$, $R \circ R \circ R$, $R \circ S \circ R$. 10

c) For any two sets A and B, show that $A - (A \cap B) = A - B$. 5

14. a) Prove that if $n \geq 1$, then $1(1!) + 2(2!) + ... + n(n!) = (n+1)! - 1$. 5

b) Show that countable union of countable sets is countable. 5

c) Let 'N' be the set of natural numbers. 'R' is a relation defined on the set $N \times N$ of ordered pairs defined by (a, b) R (c, d) if ad = bc. Prove that 'R' is an equivalence relation. 10

Module – III

15. a) Show that the set 'N' of natural numbers for the composition $a \circ b = a + b + ab$ is a semigroup. Is it a Monoid ? 8

b) Prove that every subgroup of an abelian group is a normal subgroup. 5

c) Show that if $a, b \in G$, then $(ab)^2 = a^2b^2$ if 'G' is abelian. 7

16. a) State and prove Lagrange's theorem. 10

b) Discuss different algebraic systems with two binary operations with examples. 10

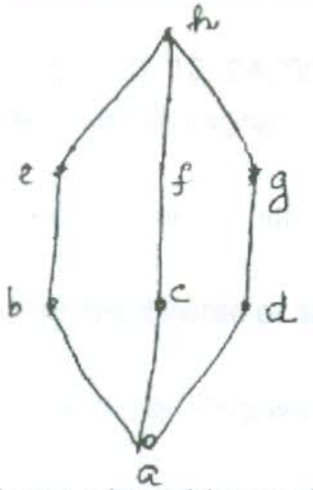
Module – IV

17. a) Show that every chain is a distributive lattice. 8

b) Differentiate between a Boolean function and Boolean expression. 2

c) Explain connected, disconnected and strongly connected graphs using examples. 10

18. a) Find all sunlattices of the given lattice. 10



b) Let x, y be arbitrary elements in a Boolean algebra (B, +, ., 1, 0, 1). Prove the Demorgan's laws.

$(x+y)' = x'y'$ and $(xy)' = x' + y'$ 10